

# On gauge unification in Type I/I' models

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## *Abstract*

We discuss whether the (MSSM) unification of gauge couplings can be accommodated in string theories with a low (TeV) string scale. This requires either power law running of the couplings or logarithmic running extremely far above the string scale. In both cases it is difficult to arrange for the multiplet structure to give the MSSM result. For the case of power law running there is also enhanced sensitivity to the spectrum at the unification scale. For the case of logarithmic running there is a fine tuning problem associated with the light closed string Kaluza Klein spectrum which requires gauge mediated supersymmetry breaking on the “visible” brane with a dangerously low scale of supersymmetry breaking. Evading these problems in low string scale models requires a departure from the MSSM structure, which would imply that the success of gauge unification in the MSSM is just an accident.

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# 1 Introduction

Perhaps the most persuasive indication of structure “Beyond the Standard Model” comes from the observation that the strong, and electroweak gauge couplings unify in the simplest supersymmetric extension of the Standard Model. Computing the radiative corrections to the gauge couplings coming from the virtual states of the MSSM one finds the  $SU(3)$ ,  $SU(2)$  and  $U(1)$  couplings become very nearly equal at the scale  $(1-3) \cdot 10^{16}$  GeV provided one adopts the  $SU(5)$  normalisation of the  $U(1)$  factor. A measure of the accuracy of this prediction may be obtained by using the assumed unification to predict the value of the strong coupling given the weak and electromagnetic couplings. One finds  $\alpha_3(M_z) = 0.126 \pm 0.003$  to be compared with the experimental value [1]  $\alpha_3(M_z) = 0.119 \pm 0.002$ . Only slightly less impressive is the fact that the SUSY radiative corrections, assuming the same unification scale, lead to a prediction of the bottom mass in the range  $3.12 < m_b(M_z) < 3.76 \text{ GeV}$  to be compared to the range  $2.72 < m_b(M_z) < 3.16 \text{ GeV}$  obtained from the pole mass  $M_b = 4.8 \pm 0.2 \text{ GeV}$  [2]. Again this prediction assumes the  $SU(5)$  prediction for the equality of the bottom to the tau mass at the unification scale. It is largely on the basis of these successes and the fact that supersymmetry is needed to solve the mass hierarchy problem that there has been so much interest in supersymmetric models and in compactification schemes that preserve a low energy supersymmetry.

Weakly coupled heterotic string compactifications accommodate these features in a very natural way. The string and gauge unification scales are predicted to be very large, of the order of the Planck scale. The  $SU(5)$  normalisation of the  $U(1)$  factor emerges for a wide variety of compactifications even without a stage of Grand Unification below the Planck scale. The relation of the bottom to tau masses is more model dependent but, because there is an underlying  $E_6$  symmetry above the compactification scale, it is easy to find specific models which do predict equality even without a stage of Grand Unification.

This picture has been challenged by the realisation that it is possible to build (open) string theories in which the string scale is much smaller [3]-[4], possibly no more than the TeV scale which might be small enough to avoid the gauge hierarchy problem. Associated with this is the possibility that there are new large dimensions, possibly as large as a millimetre. At first sight such theories seem to be inconsistent with the MSSM unification of couplings which requires a very large unification scale. However there are several possible ways that gauge unification may be maintained. One is that some, or all, of the Standard Model states may propagate in new large dimensions, leading to power law running of the gauge couplings and the possibility that unification is achieved at a much lower scale. A second possibility [5] is that the normal logarithmic running applies but the ultraviolet cut-off scale is not the string scale but is associated with a very heavy Kaluza Klein or winding mode. This is a particularly interesting possibility because the cut-off scale is related to the *infra-red* properties of the transverse (closed string) channel. The mass,  $m_{KK}$ , of the lightest Kaluza Klein states in this channel is inversely proportional to the new large compactification radius and is of  $O(10^{-4} \text{ eV})$  for a compactification radius of  $O(1 \text{ mm})$ . A state of this mass corresponds to a cutoff scale in the open string channel of  $O(M_s^2/m_{KK})$  and if one sets the string scale,  $M_s$ , to 1 TeV one obtains  $10^{17} \text{ GeV}$  for the ultra violet cut-off. This is tantalisingly close to the gauge unification scale in the MSSM.

In this letter we will consider both these possibilities for the case of Type I/I' string compactifications. We will concentrate on the question whether it is possible to maintain the remarkable success of the MSSM predictions in the sense that the running of the gauge couplings is governed by the MSSM beta functions. Our conclusion is somewhat pessimistic suggesting that MSSM unification with a low string scale does not follow as nicely as it did in the heterotic string case. Of course it may be that nature is perverse and that the success of unification in the MSSM could be just an

accident. As an illustration of this we also consider a promising Type I string compactification in which gauge unification can be made to work.

## 2 Type I/I' models and unification with low string scale

In Type I (Type I') strings at weak coupling the Standard Model fields are described by open strings confined to a  $D_p$  brane with Kaluza Klein modes with respect to  $(p - 3)$  compact dimensions in the brane and winding modes with respect to  $(9 - p)$  dimensions orthogonal to the brane. Closed strings mediate the gravitational interaction and are free to propagate in the full ten dimensional space-time. Upon compactification to four dimensional space-time, the following relations emerge between the volume<sup>3</sup>  $v_p$  of the compact dimensions of the SM brane, the volume  $v_o$  of the dimensions orthogonal to it and the string scale  $M_I$

$$M_I \sim g^2 M_P \left( \frac{v_p}{v_o} \right)^{\frac{1}{2}} \quad (1)$$

so the string scale may be reduced [6] and may be very low ("TeV scale") [4] if the transverse space is large [3]. It should however be noted that lowering the fundamental scale does not, by itself, solve the hierarchy problem for now we must explain why  $v_o \gg v_p$ . In addition to the relation of eq.(1) the string coupling is related to the four dimensional gauge coupling

$$\lambda_I \sim g^2 v_p \quad (2)$$

where  $v_p$  should be of order unity to keep  $\lambda_I < 1$ .

Particularly interesting is the case in which the MSSM is embedded in a  $D3$  brane, for in this situation one may have extra (transverse) dimensions as large as  $\leq mm$  [3] (not ruled out by experiment) while the Standard Model field has only winding modes corresponding to the large transverse dimensions. In the case the MSSM is embedded in a  $D_p$  brane with  $p > 3$  the  $p - 3$  additional dimensions cannot be much larger than  $1 TeV^{-1}$  due to the presence of Standard Model Kaluza Klein modes. However in asymmetric compactifications some or all of the  $(9 - p)$  orthogonal dimensions may be much larger.

Unlike Grand Unification, string theories predict the gauge unification scale. In the heterotic string the Standard Model fields are closed string states and the ultraviolet cutoff scale is determined by the geometry of the string world sheet. At one loop this is the torus and one may readily verify that the cutoff is at the string scale [7, 8]. In the case the Standard Model fields are described by open strings however the world sheet geometry does not regulate the divergences the latter only being eliminated after summing over the possible geometries. After eliminating the divergences the UV behaviour in the open string channel is determined by the IR behaviour in the transverse open string channel. This follows due to the closed string/open string duality and leads to alternative, but equivalent, descriptions of the behaviour of the radiative corrections to gauge couplings. The alternative open and closed string descriptions are summarised in Table 1.

Looking at the first two entries in Table 1 one may see that in the open string channel the running of the couplings is due to the familiar radiative corrections to the gauge couplings while in the closed string channel it is due to the modification of the vacuum expectation value of the bulk field,  $\phi$ , which is the coefficient of the gauge kinetic term. The latter running is due to the  $\phi$  propagator in the bulk, emitted by distant branes, evaluated at the transverse position of the SM brane. This provides

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<sup>3</sup>in Type I/I' string length units.

	Open string channel	Closed string channel
Coupling	Usual gauge coupling	v.e.v. of bulk field $\phi$ at $x_\perp$
Running	Propagation of SM nonsinglet states	$\phi$ propagator in bulk
Logarithmic; $\alpha^{-1} \sim \ln(\Lambda R)$	D=4 propagation	$d_\perp = 2$ propagation
Linear: $\alpha^{-1} \sim (\Lambda R)$	D=5 propagation. Single KK tower.	$d_\perp = 1$ propagation
Quadratic $\alpha^{-1} \sim (\Lambda R)^2$	D=6 propagation. Double “in-brane” KK tower	$d_\perp = 0$
String cut-off	UV, $M_s$ or $M_{winding} \geq M_s$	IR, Lightest $\phi$ .
Coupling constant ratios	Gauge symmetry	Geometric symmetry in the brane sector

Table 1: Alternative description of gauge coupling running in open/closed string channels.

a classical interpretation of the quantum effects on the gauge couplings (UV behaviour) on the SM brane. Logarithmic evolution in the direct channel is due to the radiative corrections involving the propagation of virtual Standard Model states in four dimensions. In the transverse channel this is due to  $\phi$  propagation in two transverse dimensions. If the open string states propagate in more than four (“in-brane”) dimensions the couplings will run like a power rather than a logarithm.

From the four dimensional point of view an (“in-brane”) extra dimension shows up as a Kaluza Klein (KK) tower of excitations which each contribute additional logarithmic running to the coupling which sum to a (linear) power once sufficient KK states can be excited. In the transverse closed string channel this corresponds to  $\phi$  propagation in one transverse dimension (the Green functions fall as  $r^{2-d_\perp}$  for  $d_\perp$  transverse dimensions). Similarly if there are two KK towers of excitations in the direct channel the couplings evolve quadratically. In the closed string channel this corresponds to the case  $d_\perp = 0$ . In the weakly coupled heterotic string with orbifold or Calabi Yau supersymmetric compactifications from 10 to 4 dimensions it might seem there could be running proportional to  $r^6$ . However, the massive KK modes giving rise to the extra dimensions fill out  $N = 2$  and  $N = 4$  supermultiplets. The latter do not contribute to radiative corrections and so the power law running is limited to  $r^2$ . In the transverse channel this result follows simply from the observation that  $d_\perp = 0$  is the limiting case.

The prediction of gauge unification also requires that the tree level coupling constant ratios be predicted. The gauge couplings are related to the dilaton couplings to the various gauge kinetic terms. In the heterotic string the reason for unification is the presence of a single dilaton v.e.v. which is the string coupling at the unification scale. For level-1 Kac-Moody theories one has the usual  $SU(5)$  relations. More generally such a relation may come from an underlying Grand Unified symmetry above the compactification scale. In Type I/I' theories it has been suggested that in the closed string channel [9] it is an underlying geometric symmetry of the brane sector that is responsible for relating the couplings.

Finally string theories predict the gauge unification scale. The prediction for the tree level ratios apply at the scale at which the radiative corrections are cut-off. As discussed above this is determined by the lightest relevant state in the transverse channel. This turns out either to be the string scale or the lightest KK scale depending on which sector of the theory one is considering [11].

## 2.1 Power-law unification

The first discussion [12] of the possibility of a low string and unification scale following from power law running employed an effective field theory approach. In this the individual logarithmic corrections of Kaluza Klein modes (with respect to  $\delta = (p-3)$  ( $p \geq 3$ ) “in-brane” dimensions) below the string scale are summed [13, 8]. This leads to the following (one-loop) RGE evolution [12]

$$\alpha_i^{-1}(\mu) = \alpha^{-1}(\Lambda) + \frac{\bar{b}_i}{2\pi} \sum_{|\vec{m}| \leq \Lambda/\mu_0} \ln \frac{\Lambda}{\mu_0 |\vec{m}|} + \frac{b_i}{2\pi} \ln \frac{\Lambda}{\mu} \quad (3)$$

$$\approx \alpha^{-1}(\Lambda) + \frac{\bar{b}_i}{2\pi} \left\{ \frac{\pi^{\delta/2}}{\delta \Gamma(1 + \delta/2)} \left[ \left( \frac{\Lambda}{\mu_0} \right)^\delta - 1 \right] - \ln \frac{\Lambda}{\mu_0} \right\} + \frac{b_i}{2\pi} \ln \frac{\Lambda}{\mu} \quad (4)$$

which shows explicitly [8] the origin of the “power-law” behaviour in the high degeneracy of Kaluza-Klein modes. Here  $\bar{b}_i$  is the  $N = 2$  beta function of KK sector and  $b_i$  is the one loop contribution,  $\delta$  is the effective number of (“in-brane”) extra dimensions,  $\Lambda$  is the cut-off scale and  $\mu_0$  is the mass of the lowest KK excitation. Although the number of extra dimensions may be as large as six, as discussed above, we expect at most quadratic running,  $\delta = 1, 2$ . The constraint (2) restricts the in-brane volume to values close to unity in string length units. In general the power-law running takes place over a very small range of energy and thus the radii ( $1/\mu_0$ ) are close to the string scale and (2) may be satisfied by symmetric unifications with the remaining  $6 - \delta$  dimensions also of order the string length scale. In the effective field theory approach unification occurs at the cutoff scale  $\Lambda$  and this scale must be identified. The usual assumption is to identify it with the string scale,  $M_I$ . However, following the discussion above, in Type I' theories it may be one should identify  $\Lambda$  with the larger winding mode mass. The correct identification depends on the details of the string theory - in some cases the winding mode excitations fill out complete  $N = 4$  multiplets and in this case the lower string scale is the appropriate choice [11]. In either case the power law running can lead to unification at a scale which may be smaller [12] than the ordinary MSSM unification scale.

While power law running can reduce the unification scale there are two difficulties in obtaining the usual MSSM predictions. The first is the sensitivity power law running introduces to the thresholds at the unification scale [14]. This is readily seen from eqs.(3), (4) where, for a low (TeV) unification scale  $\mu_0(\partial\alpha_i^{-1}/\partial\mu_0) \simeq \delta(\alpha_i^{-1}(\mu) - \alpha^{-1}(\Lambda))$ . This is to be compared to the case of normal logarithmic running in the MSSM where  $\mu_0(\partial\alpha_i^{-1}/\partial\mu_0) \simeq (\alpha_i^{-1}(\mu) - \alpha^{-1}(\Lambda))/\ln(\Lambda/\mu)$ . Thus the sensitivity to the unification scale is roughly enhanced by a factor  $\delta \ln(\Lambda/\mu)$  which is  $\geq 32$ . A more careful analysis gives  $\delta\mu_o/\mu_o = 1/224$  [14]. This means that in order to make a precision prediction for the gauge coupling one must know the detailed spectrum of the states at the unification scale. Although this does not rule out the possibility of low scale unification through power law running, it does make the success of the detailed prediction of the gauge couplings in the MSSM seem surprising.

The second difficulty one encounters in trying to duplicate the MSSM results via power law running comes from the fact that in eq.(3) the rate of running is controlled by the coefficients  $\bar{b}_i$  which are determined by the  $N = 2$  sector of the theory. The difficulty is that these are not expected to be proportional to the  $N = 1$  beta functions and thus we may expect substantial deviations from the MSSM predictions. To see this note that if one has towers of KK states for the entire MSSM spectrum,  $\bar{b}_i = -2T_i(G) + 2\sum_\psi T_i(\psi)$  which is not proportional to  $b_i^{MSSM} = -3T_i(G) + \sum_\psi T_i(\psi)$  of the ( $N = 1$ ) MSSM sector. Instead we have  $\bar{b}_i = 2b_i^{MSSM} + 4T_i(G)$ . To avoid this we should look for a model for which

$$\bar{b}_i = Kb_i^{MSSM} + C \quad (5)$$

with  $K$  and  $C$  some gauge group independent constants. In this case eq.(3) gives the same value for  $\alpha_3(M_z)$  as in the MSSM at one loop level (in such a case we would have an explanation for the “accidental” success of the MSSM gauge unification predictions).

Attempts to obtain a spectrum satisfying this condition have been made by Kakushadze [15] by including further KK states which, together with the KK excitations of the MSSM, have one loop coefficients which satisfy eq.(5). A simple example with  $K = 1$  [15] was given by a  $Z_2$  orbifold model which breaks an underlying  $N = 2$  supersymmetry to  $N = 1$ . The  $N = 2$  spectrum is that of the MSSM KK excitations together with those of additional superfields  $F_\pm$ ,  $SU(3) \times SU(2)$  singlets with  $U(1)_Y$  charge  $\pm 2$  respectively. However the addition of the  $F_\pm$ , while leading to a the  $N = 2$  beta function which satisfies eq.(5), does not reproduce the  $N = 1$  MSSM spectrum because it contains additional *light*  $N = 1$  fields with the quantum numbers of  $F_\pm$ . As a result one finds

$$\alpha_3^{-1}(M_z) = \alpha_3^{o-1}(M_z) + \frac{1}{2\pi} \frac{6}{7} \ln \frac{M_s}{M_F} \quad (6)$$

Here  $\alpha_3^0(M_z)$  is the MSSM (one loop) value while the last term is the (one-loop) contribution of  $F_\pm$  states to the running of the gauge couplings. This term depends on the (unknown) ratio  $M_s/M_F$  and thus eq.(6) makes no prediction for  $\alpha_3(M_z)$  following from the unification condition. A detailed mechanism or further input is needed to fix the value  $M_s/M_F$ . Thus, although the model may allow the presence of a low (TeV) string scale, it does not *predict*  $\alpha_3(M_z)$ . It may be that one may construct models of this type which do not modify the  $N = 1$  spectrum but one may see from this example the difficulty in accommodating the MSSM predictions in the framework of power-law running.

## 2.2 Logarithmic unification

We turn now to the possibility that the gauge couplings unify with logarithmic running. A Type I/I' full string calculation for the threshold corrections to the gauge couplings was performed for the case of an  $N = 2$  orientifold based on the  $T^4/Z_2$  orbifold [16, 17] and other  $N = 2$  orientifolds obtained by toroidal compactification of six-dimensional vacua [11]. These results were also generalised [11] to four dimensional compactifications with  $N = 1$  supersymmetry. In the latter case the cut-off for  $N = 2$  sector is not the string scale itself but the first winding mode above the string scale associated with the (“off-the-brane”) two-torus while for the light states  $N = 1$  the cut-off is set by the string scale itself. Two possibilities exist for the RGE behaviour [11, 5] at the high scale in this case. One of these is the linear “power-law” regime, corresponding to an “asymmetric” two-torus compactification, while the other has only logarithmic dependence on  $R$  (and ultimately on  $M_P$ ) in the RGE. We will now concentrate on the second case as the first one was discussed above.

The interest for phenomenology in these models is two-fold. Firstly, the large transverse volume and low string scale brings the possibility (e.g. for the case the MSSM is confined to a D3 brane) of new gravitons and their Kaluza Klein towers in the  $mm^{-1}$  range [3], a situation not ruled out by the experiment. Secondly, the mild logarithmic dependence of the couplings on the high scale raises the possibility [5] of gauge coupling unification at a large scale above  $M_I$  given by the string cut-off (first winding mode). This may be just the original unification scale of the MSSM thus elegantly providing an alternative (Type I) interpretation of the successful MSSM predictions.

Turning to a more detailed discussion of the latter possibility note that the corrections to gauge couplings come only from the  $N = 1$  (massless) sector and from the  $N = 2$  massive winding (Type I') (or Kaluza Klein, Type I) sector. Let us consider the  $N = 2$  sector first. The massive  $N = 2$  threshold corrections to  $\alpha^{-1}$  at the string scale  $M_I$  have the form [11] (for both  $\mathcal{A}$  (annulus) and  $\mathcal{M}$

(Möbius strip) geometries relevant at one loop)

$$\Delta_i^{Type I} = -\frac{1}{4\pi} \sum_m \bar{b}_{i,m} \left\{ \ln \left( \sqrt{G_m} Im U_m M_I^2 |\eta(U_m)|^4 \right) \right\} \quad (7)$$

Here  $G_m$  is the metric on the torus<sup>4</sup>  $T^m$ , and  $\sqrt{G_1} = \tilde{R}_1 \tilde{R}_2$  (for a rectangular torus). The behaviour of the thresholds<sup>5</sup> when  $Im U = \tilde{R}_1/\tilde{R}_2$  is of order one is indeed logarithmic,  $\Delta_a \sim \ln(\tilde{R}_1 \tilde{R}_2)$ .

We note that unlike the weakly coupled heterotic case [7] where quadratic dependence on the common radius of the two torus (associated with the  $N = 2$  sector) exists, in the present case this does not happen. The condition of global tadpole cancellation ensures that after adding the contributions (each regulated by an infrared cut-off) of massless closed string states emitted into the transverse channel, the two  $R^2$  dependent results obtained for  $\mathcal{A}$  and  $\mathcal{G}$  cancel. Therefore there is no quadratic term present in (7). In this calculation the ultraviolet (would-be quadratic) behaviour in the open channel is regulated by the infra-red regime of the closed string channel (c.f. Table 1).

From eq.(7), after adding the usual  $N = 1$  one loop term proportional to  $b_i$  we have

$$\alpha_i^{-1}(M_z) = \alpha_I^{-1} + \frac{b_i}{2\pi} \ln \frac{M_I}{M_z} + \frac{\bar{b}_i}{2\pi} \ln \frac{\tilde{\mu}}{M_I} \quad (8)$$

where we have taken  $\tilde{R}_1 = \tilde{R}_2$  and  $\tilde{\mu} = 1/(\tilde{R}_1 |\eta(U_m)|^2)$ . Here  $\bar{b}_i$  is the  $N = 2$  one-loop beta function coefficient.

From this equation we can immediately identify a problem in obtaining the usual MSSM results with a low string scale. As in the case of power law running, the dominant evolution is governed by the  $N = 2$  sector. We therefore have a similar difficulty to that discussed above in obtaining the MSSM results because in general  $\bar{b}_i$  is not proportional to  $b_i^{MSSM}$ . In ref.[9] an orbifold projection was used to break  $N = 2$  supersymmetry to  $N = 1$ . In this case the running up to the unification scale (i.e. first winding mode scale  $\tilde{\mu}$ ) is indeed given by the  $N = 1$  sector apparently solving this problem. However the construction works only for modding by freely acting groups and this implies the  $N = 1$  matter fields giving rise to  $\bar{b}_i$  originate from the adjoint representation of the underlying gauge group before modding. Although in the orbifolded model there are  $N = 1$  matter fields transforming as the fundamental representations  $F$  under one of the group factors, there are also fields transforming as  $\bar{F}$  [18]. This does not allow for an identification of these fields with the MSSM matter fields. As a result we still have the problem of explaining why the  $\bar{b}_i$  should satisfy eq.(5). If we assume however that this problem can be solved, eq.(8) together with eq.(5) reproduces the MSSM unification at one loop level with  $(\tilde{\mu}/M_I)^K M_I \approx 10^{16}$  GeV. For the case  $K = 1$  the unification occurs at  $\tilde{\mu} \approx 10^{16}$  GeV.

Somewhat surprisingly logarithmic unification also has a fine tuning problem. Naively our previous discussion suggests logarithmic unification will be no more sensitive to threshold effects than the MSSM. However this is misleading because here the string cut-off scale  $\tilde{\mu} \approx 10^{16}$  GeV corresponds to a cross channel exchange state with mass  $M_I^2/\tilde{\mu}$ . For a 1 TeV string scale this is  $10^{-10}$  GeV and corresponds to the lightest mass of Kaluza Klein modes in the closed string channel. However the state controlling the UV behaviour of the open string channel has vacuum quantum numbers. Such a state has no symmetry (local gauge symmetry etc) to protect it from receiving contributions to its mass at scales below the supersymmetry breaking scale. We therefore expect that such a state will acquire mass from supersymmetry breaking effects. The magnitude of such a mass depends on the supersymmetry breaking mechanism but has a lower bound governed by the supersymmetry

<sup>4</sup> We consider in the following only compactification on a single torus,  $T^1$ .

<sup>5</sup> For Type I' case we should take  $R = 1/\tilde{R}$

breaking communicated by gravitational effects. If supersymmetry is broken in a hidden sector (e.g. on another brane) and communicated to the visible sector by gravitational effects then we know such effects must be of  $TeV$  scale otherwise some of the SUSY partners of the SM should have been seen. However the supersymmetry breaking in the bulk must be at least of  $TeV$  scale as well because, in this case, SUSY breaking is communicated to the visible sector via the bulk. As a result logarithmic running is *not* a viable mechanism for gauge unification because now the closed string state mass ( $M_I^2/\tilde{\mu}$ )  $\approx 1TeV$  so with  $M_I \approx 1TeV$  we have  $\tilde{\mu} \simeq 1TeV$  instead of the  $10^{16}$  GeV needed for unification. If, to avoid this problem, one requires the closed string exchange state with mass  $M_I^2/\tilde{\mu}$  be  $\geq TeV$  scale, while maintaining high scale unification ( $\tilde{\mu} \approx 10^{16} GeV$ ) one finds the string scale is bounded by  $M_I \geq 10^{10}$  GeV.

The only possibility to maintain logarithmic unification with a low string scale is that supersymmetry breaking in the visible sector is much larger than in the bulk. This happens in gauge mediated scenarios where the supersymmetry breaking is communicated to the visible sector by gauge interactions much stronger than gravity. Such a scheme requires that supersymmetry breaking occurs on the SM brane itself. However, even in such a scheme there is a problem in keeping the string scale as low as  $1TeV$  as gravitational effects still generate a supersymmetry breaking mass of order the gravitino mass for fields in the bulk. The gravitino mass is of order  $M_{Susy}^2/M_P$  where<sup>6</sup>  $M_{Susy}$  is the supersymmetry breaking scale on our brane. Demanding that the gravitino mass is less than  $10^{-10}$  GeV to avoid the fine tuning problem implies  $M_{Susy} \leq 10^4 GeV$ . Provided one remains in the perturbative domain this is probably too small for a viable gauge mediated scheme because SUSY breaking in the visible sector is given by  $\gamma M_{Susy}^2/M$  where  $\gamma$  is the effective coupling of the visible sector to the SUSY breaking sector and  $M$  is the messenger mass ( $M \geq M_{Susy}$ ). In viable models it occurs at one loop order for gauginos [19] suggesting  $\gamma$  is too small to generate acceptable masses for the MSSM sparticle spectrum. Even if it proves to be possible one sees that the resulting model is heavily constrained requiring gauge mediated supersymmetry breaking with the gauge mediator mass,  $M$ , in the  $TeV$  range too. Gauge mediation with a light messenger sector ( $< 10^5 GeV$ ) is however unnatural [20].

To summarise, in both the case of power law and logarithmic running there are difficulties in accommodating the success of the MSSM unification predictions in models with a low string scale. In the case of power law unification there is also an enhanced sensitivity to the details of the mass spectrum at the unification scale making the success of the detailed prediction of the gauge couplings in the MSSM quite surprising. In the case of logarithmic running there is a fine tuning problem which can be alleviated by raising the string scale significantly above the  $TeV$  scale. However in both cases it is difficult to generate the MSSM beta function via the  $N=2$  sector. To avoid this difficulty in these models it is thus necessary to give up the MSSM structure. From this point of view, in this case the MSSM remarkable success in predicting gauge unification appears to be accidental. To illustrate this last possibility we consider unification in a promising Type I which has a multiplet structure close to that of the MSSM.

### 2.3 A Type I model with intermediate unification

The model we wish to consider is a specific example of a larger class of  $D = 4$  Type I vacua obtained from a standard  $D = 4$ ,  $N = 1$  compact Type IIB orientifold with  $D_p$  branes and anti- $D_p$  branes located at different points of the underlying orbifold [21]. We consider here the class of models of [21] which have gravity mediated supersymmetry breaking with the full Standard Model embedded in a 9-brane sector and with an additional set of 5-branes trapped at the fixed points of the  $Z_3$  orientifold.

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<sup>6</sup>We denote by  $M_P$  the Planck scale.



This model has no  $N = 2$  sector and the UV cutoff scale for the  $N = 1$  sector is the string scale [11]. As we shall discuss this model can give gauge unification via the radiative corrections of the  $N = 1$  sector with an intermediate string scale and without introducing significant threshold sensitivity. The price to achieve this is to depart from the MSSM light matter content and to start with a non-MSSM relation between the gauge couplings. In particular the normalisation of the hypercharge of these states is  $\alpha_Y/\alpha_s = 3/11$  at the string scale (instead of the usual “SU(5) factor”  $3/5$ ). The massless spectrum contains the spectrum of the MSSM. In addition there are five further pairs of Higgs doublets and three vector-like right handed colour triplets ( $d^c + \bar{d}^c$ ). The remaining states of the model may have mass of the order of string scale. In determining the unification prediction of this model we follow [21] and assume that these additional states have a common *bare* mass<sup>7</sup>,  $\tilde{m}$ , which must not be greatly different from the electroweak scale.

Including the threshold effects at  $\tilde{m}$ , the two loop RGE equations are given by

$$\alpha_i^{-1}(M_z) = -\delta_i + \alpha_s^{-1} + \frac{b_i}{2\pi} \ln \left[ \frac{M_s}{M_z} \right] + \frac{\delta b_i}{2\pi} \ln \left[ \frac{M_s}{\tilde{m}} \right] + \frac{1}{4\pi} \sum_{j=1}^3 Y_{ij} \ln \left[ \frac{\alpha_g}{\alpha_j(\tilde{m})} \right] + \frac{1}{4\pi} \sum_{j=1}^3 \frac{b_{ij}}{b_j} \ln \left[ \frac{\alpha_s}{\alpha_j(M_z)} \right] \quad (9)$$

where

$$Y_{ij} = \frac{\delta b_j}{b_j b'_j} [2b_j T_j(G) \delta_{ij} - b_{ij}] \quad (10)$$

with  $T_j(G) = \{0, 2, 3\}_j$ ,  $b_{ij}$  is the two loop beta function as in the MSSM but with  $3/11$  hypercharge normalisation,  $b_j = \{11\xi, 1, -3\}_j$ , ( $\xi = 3/11$ ),  $b'_j = b_j + \delta b_j$ , with  $\delta b_j = \{7\xi, 5, 3\}$  to account for the additional five Higgs pairs and three pairs of right handed colour triplets. For the terms  $Y_{i3}$  equations (9) should be understood as *limits* of  $b'_3 \rightarrow 0$  which indeed give the appropriate (finite) results when multiplied by the term  $\log(\alpha_g/\alpha_3(\tilde{m}))$ . Finally one should include the low energy supersymmetric thresholds  $\delta_i$  and regularisation scheme conversion factors ( $\overline{MS} \rightarrow \overline{DR}$ ). Usually  $\delta_i$ 's are included as an overall effect on  $\alpha_3(M_z)$  through an *effective* (low-energy) supersymmetric threshold [22]. In this case the different hypercharge normalisation of  $\delta_1$  prevents us from using this approach, although their overall effect on  $\alpha_3(M_z)$  is not expected to change significantly from the “normal” hypercharge case (when  $\alpha_3(M_z)$  is reduced by an amount of  $\approx 2 - 4\%$ ).

The predictions for the string coupling and the string scale  $M_s$  can be read from the plots in Figure 1 to see that the model indeed allows for a logarithmic unification at about  $\approx 10^{10} M_z$  GeV for a strong coupling consistent with the experimental value [1]  $\alpha_3(M_z) = 0.119 \pm 0.002$ . This requires an intermediate threshold at about<sup>8</sup>  $\tilde{m} \approx (100 - 300)M_z$ .

One can also compute the bottom to tau mass ratio, taking account of the different normalisation of  $\alpha_Y/\alpha_s$ . For the model with the above spectrum set-up, we find the following one loop level result (including only gauge effects)

$$\frac{R(M_s)}{R(M_z)} = \left( 1 + \frac{\alpha_s b'_1}{2\pi} \ln \frac{M_s}{\mu_o} \right)^{\frac{1}{b'_1} \frac{2N_1 \delta b_1}{b_1}} \left( \frac{M_s}{\mu_o} \right)^{\frac{\alpha_s}{2\pi} \frac{2N_3 \delta b_3}{b_3}} \left( \frac{\alpha_s}{\alpha_1(M_z)} \right)^{-\frac{2N_1}{b_1}} \left( \frac{\alpha_s}{\alpha_3(M_z)} \right)^{-\frac{2N_3}{b_3}} \quad (11)$$

where  $N_1 = -10/9\xi$ ,  $N_3 = 8/3$  and  $R(Q)$  denotes the ratio bottom to tau mass at the scale  $Q$ . For  $\alpha_s \approx 0.1$  (needed to generate  $\alpha_3(M_z)$ ) we find that  $R(M_z) \approx 5R(M_s)$ . If we assume bottom-tau unification at the string scale, this implies  $m_b(M_z) \approx 8.731 \text{ GeV}$ . To correct this, model dependent Yukawa effects not considered here must be very significant, but then the MSSM result for the bottom-tau mass ratio should be regarded as yet another accident.

<sup>7</sup> For the purpose of two loop RGE only the bare mass of the fields is needed.

<sup>8</sup> The results are subject to additional thresholds at the string scale, which may change this picture significantly.

This example shows that it is possible in a realistic model to obtain good results for gauge unification in a manner incompatible with the original MSSM, demonstrating that the latter's success may indeed be an illusion. However it also illustrates the difficulties a non-MSSM scheme must surmount in order to approach the accuracy of the MSSM prediction. Although the unification is logarithmic the sensitivity to thresholds is considerably enhanced due to the light states additional to the MSSM. For example the uncertainty in the additional Higgs doublets mass translates to the uncertainty  $\delta\alpha_2^{-1} = (5/2\pi)\delta M_H/M_H$ . If one is to predict  $\alpha_3(M_z)$  to the present experimental accuracy then one must know these masses to better than  $\delta M_H/M_H = 0.1$ . In the light of this the precision of the MSSM prediction for the strong coupling seems even more remarkable and makes it harder for us to accept it is just an accident.

### 3 Conclusions

We have considered the possibility that Type I string theories with a low (TeV) string scale may accommodate the successful MSSM predictions for the unification of the gauge couplings. Our conclusions are pessimistic. Models with power-law running have a significant sensitivity to the unification scale thresholds. Models with logarithmic running up to the string cut-off (winding) scale far above the string scale reduce the sensitivity to this scale but have a fine tuning problem associated with the closed string spectrum which requires either a very special Supersymmetry breaking sector or an intermediate string scale. In both cases it is difficult to generate the MSSM beta function via the N=2 sector. Of course it may be that the success of the MSSM predictions is illusory and the existence of a realistic Type I theory which makes a non-MSSM prediction for the gauge couplings consistent with the measured values demonstrates this possibility. However, due to the threshold dependence on the non-MSSM states, it is not possible in this model to make a prediction for the gauge couplings to the same accuracy as the MSSM result without a detailed knowledge of the masses at the unification scale. All this emphasizes the fact that if the MSSM result is regarded as an accident it is a surprisingly accurate accident! In the light of all this it seems to us that the original heterotic string (possibly strongly coupled), being able naturally to accommodate the MSSM results, still offers the best explanation of the unification of gauge couplings.

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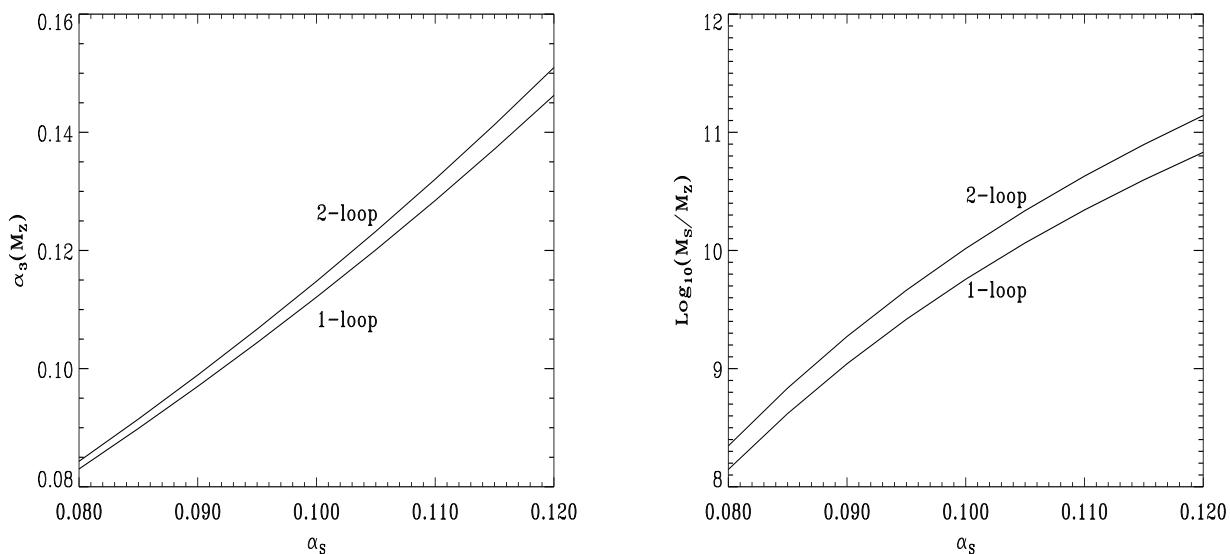


Figure 1: The values of  $\alpha_3(M_z)$  (left) and  $\log_{10}(M_s/M_z)$  (right) as a function of the string coupling  $\alpha_s$ . In both cases the lower curve is a one loop approximation while the upper curve is the two loop case. For  $\alpha_3(M_z) \approx 0.12$  we require  $\alpha_s \approx 0.1$ . The overall picture for  $\alpha_3(M_z)$  is expected to be shifted downwards by about 2 – 4% after the inclusion of the low energy threshold corrections  $\delta_i$  with the appropriate hypercharge normalisation, while the remaining plot is not expected to change significantly.

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